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Studies In Statistical Signal Processing

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STUDIES IN STATISTICAL SIGNAL PROCESSING

This is a final report on Contract AFOSR-88-0327, for the period of July 1, 1988 to June 30, 1990. Section 1 provides a brief overview of our work, while Sections 2–5 describe in some detail our recent results on efficient factorization of structured matrices, recursive layer peeling, recursive state-space synthesis and wavelet representations.

1. INTRODUCTION

The primary objective of our research is to develop efficient and numerically stable algorithms for nonstationary signal processing problems by understanding and exploiting special structures, both deterministic and stochastic, in the problems. We also strive to establish and broaden links with related disciplines, such as cascade filter synthesis, scattering theory, numerical linear algebra, and mathematical operator theory for the purpose of cross fertilization of ideas and techniques. These explorations have led to new results both in estimation theory and in these other fields, *e.g.,* to new algorithms for triangular and QR factorization of structured matrices, new techniques for root location and stability testing, new realizations for multiple-input/multiple-output (MIMO) transfer functions, and new recursions for orthogonal polynomials on the unit circle and the real line as well as on other curves.

For several years, the guiding principle in these studies has been the concept of generalized displacement structure (Lev-Ari and Kailath (1986)), which generalized and subsumed our earlier work on Toeplitz-oriented displacement structure (Kailath, Kung and Morf (1979); see also Lev-Ari and Kailath (1984)). A related notion of displacement structure has also emerged from recent work of Heinig and Rost (1984,1987) of East Germany. While they are aware of our work, and make some attempts to relate to it, their approach and methodology are significantly different from ours. In particular, they focus only on the problem of inversion of structured matrices via algebraic methods,

while our work has primarily addressed triangular factorization of such matrices, and our approach is based on a generating function characterization of matrices. The triangular factorization problem is in many senses more fundamental than inversion, and has more consequences for signal processing, linear algebra, operator theory and other fields. In fact, recently we were able to reduce the inversion problem for structured matrices to the factorization of certain block-matrices with structured blocks (see Chun and Kailath (1989)). This result also confirms and clarifies an earlier observation [see Lev-Ari and Kailath (1984)] on the relation between efficient inversion and efficient factorization of structured matrices: only some of the structured matrices that admit an efficient factorization procedure can also be efficiently inverted.

The generating function approach also suggests a natural *system-theoretic* interpretation of the theory, which allows a study of various problems in system theory, such as minimal realization, Padé approximation, control design, and a variety of root distribution (stability) problems for polynomials. Perhaps the most prominent system-theoretic aspect of our efficient factorization techniques is that they can be interpreted as recursive identification procedures for certain *lossless cascade models.* For instance, the classical Schur algorithm is also a procedure for step by step identification and 'peeling' of the layers of a transmission-line with a piecewise constant characteristic impedance [Bruckstein and Kailath (1987)]. Such layered models have been used for quite some time in oil exploration and in marine seismography. They involve two scalar signals propagating in opposite directions; consequently, the characteristics of the model can be captured by a single scalar input-scattering function. Schur's original formulation of his algorithm was, in fact, in terms of this scattering function.

We have recently begun to extend our methods to *multichannel cascade models*, which involve involve multiple signals propagating both in the forward and in the opposite (backward) direction. Since such models are represented by *matrix scattering functions*, it would seem that the corresponding layer-peeling procedures need to be rederived in

matrix (or block) form. This is certainly possible (see, *e.g.,* Delsarte, Genin and Kamp (1979)), but results in the introduction of computation-intensive matrix operations, such as the evaluation of matrix square roots. In contrast to this approach, we have succeeded in obtaining layer-peeling procedures that involve only elementary (2×2) circular and hyperbolic rotations, and therefore require only scalar computations. We have achieved this by incorporating the notion of *modular decomposition*, which has been used in the past to decompose the multichannel Levinson algorithm (see Sakai (1982)) and the deterministic recursive-least-squares lattice filter (Lev-Ari (1987)). Our modularly-decomposed filters can be implemented in pipelined parallel processing hardware (such as systolic arrays), with the throughput being independent of the number of channels (*i.e.,* the number of forward and backward signals flowing through the model).

Moreover, our modular formulation makes it possible to extend the Schur algorithm to matrix functions with poles within the unit disc, which arise in various control problems and in particular in model-order reduction with Hankel norm (see, *e.g.,* Genin and Kung (1981)). Such an extension would have been impossible in the block-formulation of Delsarte, Genin and Kamp (1979); and it would be quite difficult to obtain even in the 'tangential' formulation of Dewilde and Dym (1981). Moreover, we have applied the same approach to extend in a similar way also the Nevanlinna algorithm, which includes Schur's algorithm as a particular case (*i.e.* , all extraction points are at the origin).

While the Schur and Nevanilnna algorithms synthesize lossless cascade models from their *input scattering function*, there has been also much work done on the synthesis of lossless arrays from state-space representations (see, *e.g.,* Genin et al. (1983), Roberts and Mullis (1987)). This approach involves more computation, but results in a greater flexibility in choosing the array configuration (which need not be a cascade). In addition, it provides convenient analytical tools for the analysis of finite precision effects such as overflow oscillations and roundoff noise.

We have recently formulated a new (recursive) approach to lossless cascade synthesis

from state-space representations. Our approach not only requires significantly less computations than the traditional method (which is based on conversion to balanced form), but also leads in a natural way to a matrix-domain formulation of the *generalized Schur algorithm* for triangular factorization of structured matrices. This new formulation subsumes most previously published procedures for factorization of structured matrices and, in particular, the matrix-domain formulation of Chun and Kailath (1989, 1990) and the transform-domain formulation of Lev-Ari and Kailath (1984, 1986). Finally, the state space approach is also helpful in describing the relation between the cascade models associated with structured matrices (as described in Lev-Ari and Kailath (1986)) and the models associated with the inverses of these matrices. These 'inverse' models are the starting point for the derivation of the Levinson algorithm and the Gohberg-Semencul formula for structured matrices other than Toeplitz.

Another application of the state-space approach has been presented by Glover (1984) in the context of model-order reduction. We have recently begun to explore the possibility of his method with our results on multichannel Schur algorithms in order to obtain a computationally-efficient procedure for determining a modular realization of the reduced-order filter.

So far we have considered only (linear) models with *time-invariant parameters.* In order to be able to characterize processes with stable (*i.e.,* bounded) persistently non-stationary dynamics we need to allow models with *time-varying parameters.* For instance, processes with periodically-varying correlation (*i.e.,* $r_{t+P,\tau+P} = r_{t,s}$ for all t, s) involve models with periodically-varying coefficients. One way to overcome such complications is to consider representations of processes as linear combinations of known functions, similar to the Karhunen-Loeve representation. However, we focus on basis functions that are *independent of the statistics of the process in consideration.* In particular, we have considered basis functions derived from the theory of wavelets (see, *e.g.,* Daubechies (1988), Mallat (1989)). We have characterized the asymptotic behavior of such representations

with increasing level of resolution and have established (so far only for periodically-correlated processes) the correlation structure of the representation coefficients.

2. FACTORIZATION OF STRUCTURED MATRICES

Our early work on factorization and inversion of Toeplitz and close-to-Toeplitz matrices led us to the observation that for certain matrices the *displacement matrix*

$$\nabla_{\mathbf{Z}}\mathbf{R} := \mathbf{R} - \mathbf{Z}\mathbf{R}\mathbf{Z}^* \ , \quad \mathbf{Z} = [\delta_{i,j+1}]_{i,j=0}^{n}$$

has low rank. Notice that $\mathbf{Z}$ has unity elements on the first subdiagonal and zeros elsewhere. Consequently, the displacement matrix $\nabla_{\mathbf{Z}}\mathbf{R}$ is the difference between the matrix $\mathbf{R}$ and the matrix $\mathbf{Z}\mathbf{R}\mathbf{Z}^*$ obtained by displacing $\mathbf{R}$ one step along the main diagonal. In particular, the displacement rank (*i.e.,* the rank of $\nabla_{\mathbf{Z}}\mathbf{R}$) is 2 for both Toeplitz matrices and inverses of Toeplitz matrices. We have shown in previous work (largely supported by AFOSR) that the displacement concept is a key tool for developing fast algorithms of many kinds, including factorization and inversion of Toeplitz and near-Toeplitz matrices, as well as fast (generalized Levinson and Schur) algorithms for solving linear systems with such coefficient matrices. Not surprisingly, these results led naturally to cascade orthogonal structures for the prediction of nonstationary processes (Lev-Ari and Kailath (1984)). We have also found that the same concept is tightly connected to the more general problem of cascade filter synthesis in network theory and digital filtering as well as to a variety of inverse scattering problems (some references are Rao and Kailath, (1984, 1985), Bruckstein and Kailath (1987), Lev-Ari (1988)).

Later we extended the concept of displacement structure to a very broad family of structured matrices, including Hankel matrices and their inverses, sums of Toeplitz and Hankel matrices and several others (Lev-Ari and Kailath, (1986)). The generalized displacement of a matrix $\mathbf{R}$, is defined as $d(\mathbf{Z},\mathbf{Z})\mathbf{R}$ where

$$d(\mathbf{A},\mathbf{B})\mathbf{R} := \sum_{k,l=0}^{N} d_{k,l}\mathbf{A}^k\mathbf{R}(\mathbf{B}^*)^l \ , \tag{1}$$

and the asterisk ($*$) denotes Hermitian transpose (complex conjugate for scalars).

The concept of displacement structure and its properties are more conveniently described in terms of *generating functions.* The generating function of a matrix $\mathbf{R}$ is a power series in two complex variables, viz.,

$$R(z,w) := [1 \;\; z \;\; z^2 \;\; \ldots]\mathbf{R}[1 \;\; w \;\; w^2 \;\; \ldots]^* \tag{2}$$

The displacement $d(\mathbf{Z},\mathbf{Z})\mathbf{R}$ of a matrix has the generating function $d(z,w)R(z,w)$, where

$$d(z,w) = \sum_{k,l=0}^{N} d_{k,l} z^k (w^*)^l \tag{3a}$$

Thus the generating function of a Hermitian matrix with a displacement structure has the form

$$R(z,w) = \frac{G(z)\; J\; G^*(w)}{d(z,w)} \tag{3b}$$

where J is any constant nonsingular Hermitian matrix. The triple $\{d(z,w), G(z), J\}$ is called a generator of $R(z,w)$, since it uniquely determines the generating function $R(z,w)$.

We have extended our previous work (Lev-Ari and Kailath (1984)) on efficient factorization of matrices with displacement structure to accommodate the generalized displacement $d(\mathbf{Z},\mathbf{Z})\mathbf{R}$, and we have shown (Lev-Ari and Kailath (1986), Lev-Ari (1989)), that efficient factorization of $\mathbf{R}$ is possible if, and only if, there exist power series $\alpha(z), \beta(z)$ (with arbitrary radii of convergence) such that

$$d(z,w) = \alpha(z)\alpha^*(w) - \beta(z)\beta^*(w) \quad . \tag{4}$$

To obtain the factorization of $\mathbf{R}$ one has to propagate the recursion (with $G_0(z) \equiv G(z)$)

$$(z - \zeta_i) G_{i+1}(z) = G_i(z)\Theta_i(z) \qquad i = 0, 1, 2, \ldots \tag{5}$$

where $\Theta_i(z)$ is specified in terms of $G_i(\zeta_i)$ (see, *e.g.,* Lev-Ari and Kailath (1986), Lev-Ari (1989)). The lossless $(p+q)$-port $\Theta_i(z)$ can be decomposed into a constant (memoryless) part, which has $p+q$ inputs/outputs and a scalar (single-input/single-output) dynamic

part (Lev-Ari (1989)). This decomposition is particularly simple when $d(0,0) \neq 0$, as described in Fig. 1.

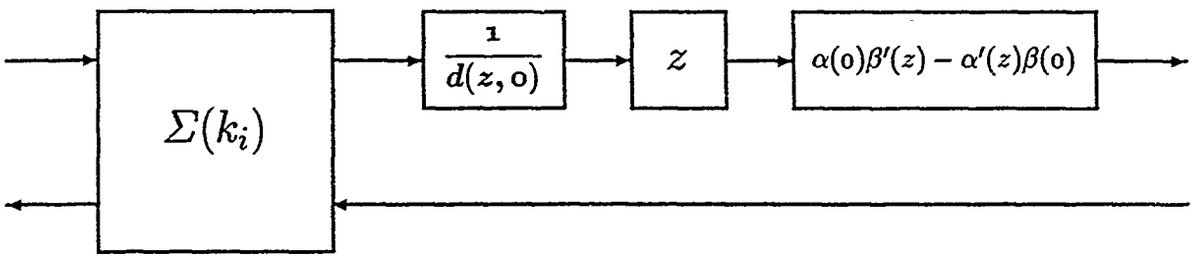


Figure 1. Decomposition of $\Theta_i(z)$ $(p = 1 = q, \zeta_i = 0)$.

The standard choice of the *extraction points* $\{\zeta_i\}$, *i.e.*, $\zeta_i = 0$, produces triangular factorizations; other choices can be useful in root-location and filter synthesis procedures (see, *e.g.,* Deprettere and Dewilde (1980), Vaidyanathan and Mitra (1984)). This algorithm requires $O(n^2)$ computations to factor a structured $n \times n$ matrix $\mathbf{R}$ in the form $\mathbf{R} = \mathbf{LDL}^*$, in contrast to the conventional LDL^* algorithm which requires $O(n^3)$ operations to factor an arbitrary $n \times n$ matrix. The i-th element of the diagonal matrix $\mathbf{D}$ and the i-th column of the lower triangular matrix $\mathbf{L}$ are determined by the coefficients of the power series expansion of $G_i(z)$.

3. RECURSIVE LAYER PEELING

The fundamental factorization procedure (5) for structured matrices, viz.,

$$(z - \zeta_i)G_{i+1}(z) = G_i(z)\Theta_i(z)$$

is, at the same time, also a *layer-peeling procedure.* Starting with $G_0(z)$, which we can interpret as boundary data for a layered medium, we identify an elementary layer with chain-scattering matrix $\Theta_0(z)$, then "peel" it off to obtain $G_1(z)$, the boundary data for the rest of the medium (with the first layer removed), and repeat the same procedure

again and again. Such layer-peeling recursions have been used in cascade filter synthesis (see, *e.g.,* Dewilde, Vieira and Kailath, (1978); Vaidyanathan and Mitra (1984)), in inverse scattering (Bruckstein and Kailath (1987)), zero-location (Lev-Ari, Bistritz and Kailath (1987)), and model-order reduction (see, e.g., Genin and Kung (1981)).

The classical work of Schur (1917) forms the basis for much of the subsequent work on layer peeling procedures. Schur's algorithm associates a sequence of so-called *reflection coefficients*, all with magnitude bounded by unity, with every passive scattering function, *i.e.,* a function $f(z)$ that is analytic and bounded by 1 in the unit disc. In particular, if $f(z)$ is an *all-pass function*, which means that $|f(e^{j\theta})| = 1$ for all θ, then Schur's algorithm produces a *finite* sequence of reflection coefficients $\{k_i \ ; \ \ 0 \leq i \leq n\}$, where $|k_n| = 1$ and $|k_i| < 1$ for $0 \leq i \leq n-1$.

Another property of the algorithm is that starting with a passive scattering function it generates a sequence of such functions. This is the essence of the layer peeling method: a single step of the Schur algorithm applied to a passive medium leaves a medium with the same property, which makes it possible to apply the same step again and again. A single step of the Schur algorithm corresponds to the removal (or peeling) of an elementary lossless two-port. Thus, the algorithm produces a discrete transmission-line model, whose input scattering function is $f(z)$ (Fig. 2).

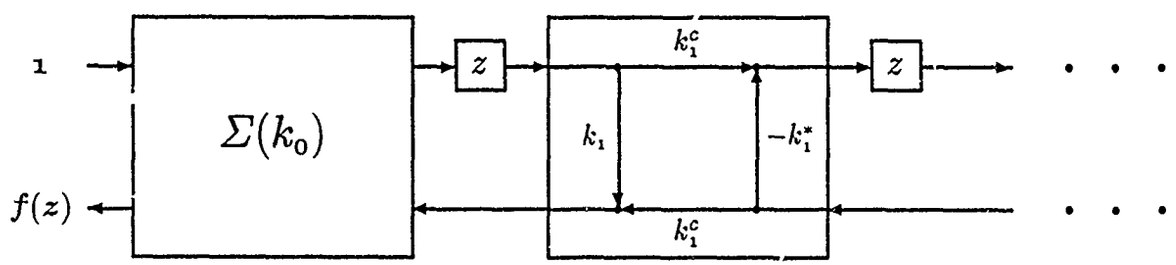


Figure 2. Transmission-line model associated with the Schur algorithm.

In addition to the recursive characterization of passivity via the constraint on the

magnitude of the reflection coefficients, Schur also introduced an operator-norm characterization of passivity: he proved that for any function $f(z)$ that is analytic in the unit disc we can construct an infinite lower-triangular Toeplitz matrix whose first column consists of the coefficients of the power series expansion of $f(z)$, viz.,

$$\mathbf{L}(f) = \begin{pmatrix} f_0 & & \\ f_1 & f_0 & 0 \\ f_2 & f_1 & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & & \cdot \quad \cdot \end{pmatrix} \tag{6a}$$

such that

$$\sup_{|z|<1} |f(z)| = \|\mathbf{L}(f)\|_2 \leq 1 \tag{6b}$$

where $\|A\|_2$ denotes the conventional norm of a matrix A, *i.e.*,

$$\|A\|_2 := \sup_x \frac{\|Ax\|_2}{\|x\|_2} \tag{6c}$$

and $\|x\|_2$ denotes the Euclidean (ℓ_2) norm of a vector x.

Schur's algorithm was later extended by Cohn (1922) to functions with *poles in the unit disc*, but only to rational "all-pass" functions, *i.e.*, to functions $f(z)$ of the form

$$f(z) = \lambda \frac{p^{\#}(z)}{p(z)} \tag{7a}$$

where $p^{\#}(z)$ denotes the conjugate reversal operation, viz.,

$$p^{\#}(z) = z^{deg\ p(z)}[p(1/z^*)]^* \tag{7b}$$

The now well-known Schur-Cohn test associates with each such function [1] a finite sequence of reflection coefficients, some of which have magnitudes larger than 1. Moreover, it has been shown (*e.g.*, using the properties of Bezoutians on the unit disc) that the number of poles of $f(z) = p^{\#}(z)/p(z)$ inside the unit-disc equals the number of singular

[1] Assuming $p(z)$ has no zeros at $z = 0$, and applying the algorithm to $f(z)/\lambda$.

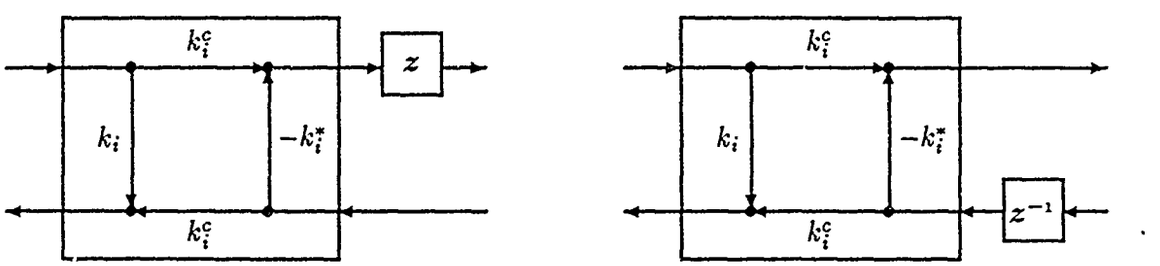
values of the matrix $\mathbf{L}(f)$ that are larger than $|\lambda|$ or, equivalently, the number of negative eigenvalues of the following finite rank matrix,

$$\mathbf{R} := |\lambda|^2\mathbf{I} - \mathbf{L}(f)\mathbf{L}^*(f) \quad . \tag{8}$$

Rational allpass functions of a given degree k are members in the family H_k^∞ of all functions with k poles (or less) inside the unit circle, and whose magnitude is bounded on the unit circle, *i.e.*,

$$\sup_{|z|=1} |f(z)| < \infty$$

It turns out that the Schur-Cohn algorithm does not map the family H_k^∞ into itself, except when $k = 0$. This means that this algorithm does not admit the same layer-peeling interpretation as the standard Schur algorithm. Nevertheless, we have found that it is possible to modify the Schur algorithm in such a way that the resulting recursion indeed maps the family H_k^∞ into itself and, therefore, admits the same layer-peeling interpretation as the classical Schur algorithm. Moreover, the layers involve only elementary (2×2) orthogonal and hyperbolic rotations (Ackner, Lev-Ari and Kailath (1990a)).



p-type layer $\quad (k_i = f_i(0))$ $\qquad$ n-type layer $\quad (k_i = 1/f_i^*(0))$

Figure 3. Layers for the modified Schur algorithm

Furthermore, our modified recursion applies to every function $f(z) \in H_k^\infty$, and not just to allpass functions. Each layer in the resulting transmission-line model has an sign

or 'polarity' (Fig. 3). While a *p-type layer* (*i.e.,* one of positive polarity) maps H_k^∞ into itself, a *n-type layer* maps H_k^∞ into H_{k-1}^∞ namely, it reduces by one the number of poles within the unit disc. Therefore, the number of n-type layers in the transmission-line model that is generated by our modified algorithm equals the number of poles that the function $f(z)$ has within the unit disc. This is the key idea in the construction of efficient procedures for zero-location and in the solution of several extension problems arising in various control applications (Ackner, Lev-Ari and Kailath (1990a)).

In the layer-peeling procedure (5) we have the freedom of choosing the extraction points $\{\zeta_i\}$. The flexibility in choosing these points enables us to optimize the number of computations in the algorithm and to overcome singularities. The standard choice $\zeta_i = 0$ corresponds to the Schur algorithm. The general choice corresponds to the Nevanlinna algorithm (Nevanlinna (1929)). In (Ackner, Lev-Ari and Kailath (1990b)) we extended our results from (Ackner, Lev-Ari and Kailath (1990a)) to the Nevanlinna algorithm and showed how to modify the recursions in the case of meromorphic functions. The computational procedure corresponding to this case involves *Newton Series expansions*, viz.,

$$f(z) = \sum_{k=0}^{\infty} f_k \Psi_k(z)$$

where

$$\Psi_k(z) := \prod_{j=0}^{k-1} (z - \zeta_j) \quad ,$$

instead of the MacLaurin (*i.e.* , power series) expansion used in the computational form of the Schur algorithm (see, *e.g.* , Kailath (1987)). Applications of the Nevanlinna algorithm include model-order reduction and solution to several other interpolation problems arising in control and signal processing.

Recursive layer peeling for a medium with multiple inputs and outputs involves a generalization of the Schur algorithm to *matrix-valued analytic* functions. One version of the matrix Schur algorithm (Delsarte, Genin and Kamp (1979)) requires hyperbolic *matrix rotations*, which are computationally expensive since they involve the finding

of the square root of (positive definite) matrices. Moreover, this approach cannot be generalized to matrix functions with elements in H_k^∞, because now it would involve square roots of indefinite matrices.

An alternative approach to the *matrix analytic* case was taken by Dewilde and Dym (1981). They extract simpler layers than in the method of Delsarte, Genin and Kamp and, as a result, there is no need to compute square roots of matrices. This computational procedure, which is known as a *tangential Schur algorithm*, was the starting point for our research on layer-peeling methods for MIMO (multiple-input/multiple-output) systems.

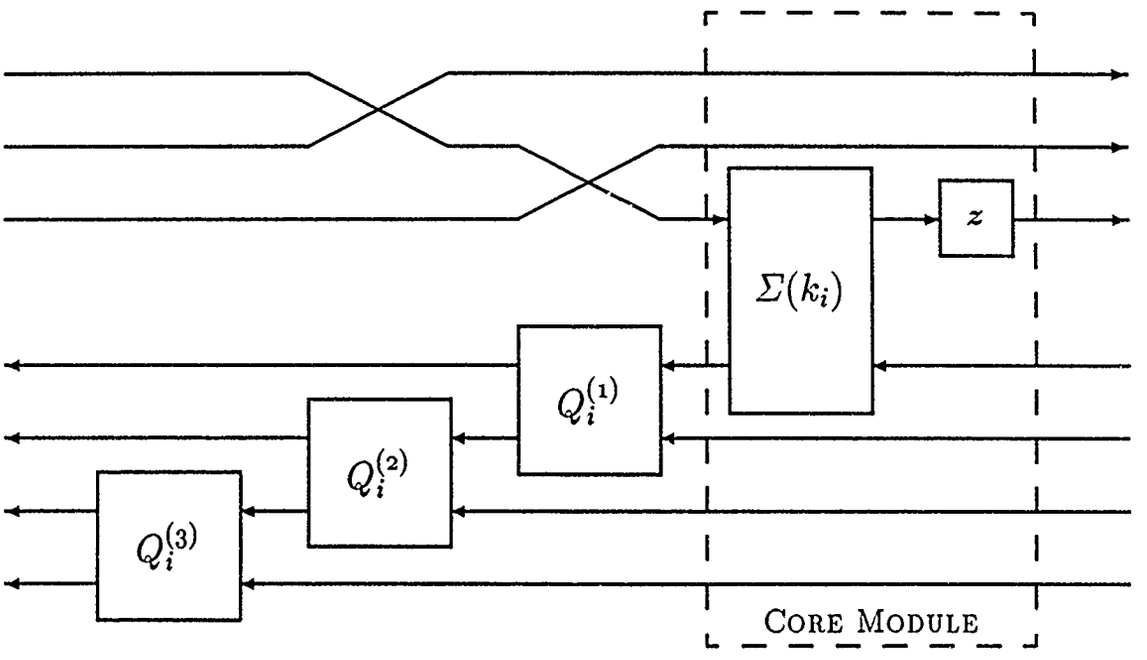


Figure 4. Single layer of the transmission-line model associated with the multichannel Schur algorithm ($p = 3, q = 4$)

We have shown that the procedure of Dewilde and Dym can be transformed into an equivalent form that involves only elementary (2×2) rotations (Ackner, Lev-Ari and Kailath (1990c)). In fact, for $p \times q$ matrix scattering functions, the peeling of each layer in our version of the algorithm is implemented by a sequence of $q-1$ elementary orthogonal rotations $\{Q_i^{(k)};\ 0 \leq k \leq p-1\}$, and a *core module*, consisting of a single-channel layer (*i.e.,* a memoryless lossless two-port with reflection coefficient k_i, and a delay) (Fig. 4). These operations can be easily implemented in either software or hardware.

In addition to requiring significantly fewer computations than the tangential Schur algorithm, our version also serves to clarify the relationship between the MIMO case and the better-known scalar or SISO (single-input/single-output) case. The MIMO procedure differs from the scalar one only by the presence of elementary orthogonal rotations. Thus both procedures share the same core module, which consists of a single elementary hyperbolic rotation and a (block-) delay element.

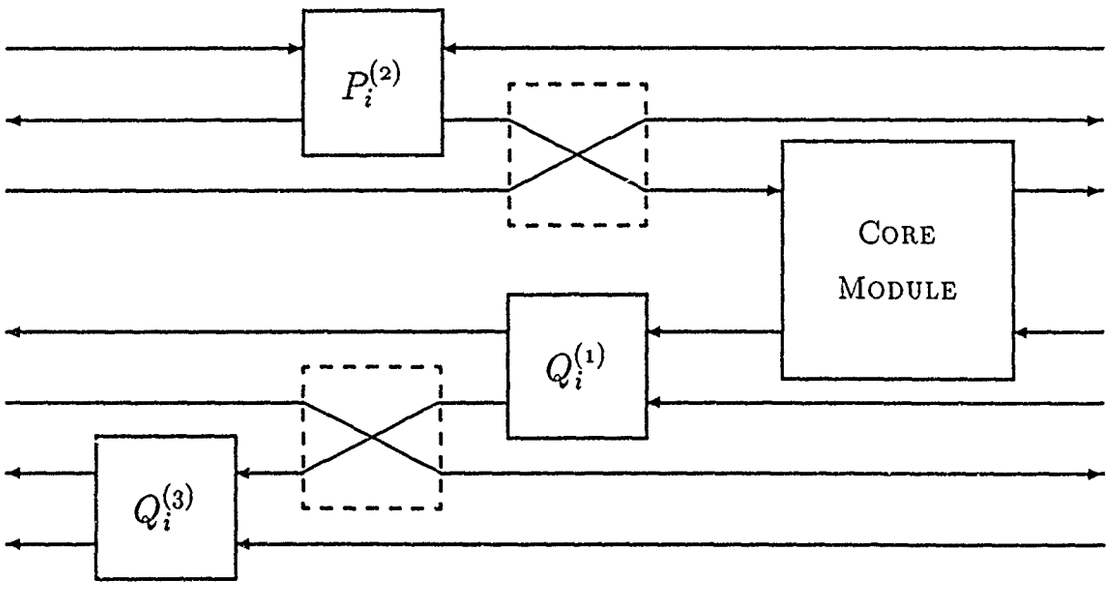


Figure 5. Structure of single layer in the non-analytic case

Consequently, we are able to show that our version of the tangential Schur algorithm can be modified to accommodate scattering functions with poles within the unit circle, and that this modification affects only the core module (*i.e.,* it is independent of the number of inputs or outputs). This means that each layer still has a 'sign' or polarity, as in the SISO case, and that the number of n-type layers coincides with the number of Smith-McMillan poles of the given matrix scattering function. Unlike the analytic case, we now have trivial cells (indicated by dashed boxes) both in the upper and lower parts of the diagram (Fig. 5). This results in a total of 2^p possible configurations for each layer, in contrast to a single configuration in the (matrix) analytic case.

A single layer of our tangential Schur algorithm involves q parameters: one in the core module and $q-1$ in the preceding orthogonal rotations. In comparison, the matrix Schur algorithm of Delsarte, Genin and Kamp (1979), viz.,

$$zF_1(z) = M_1[I - F(z)F^*(0)]^{-1}[F(z) - F(0)]M_2^{-1}$$

with

$$M_1^*M_1 = I - F(0)F^*(0) \quad , \qquad M_2^*M_2 = I - F^*(0)F(0)$$

which corresponds to p layers of the tangential algorithm, has the same number of parameters $(p \times q)$ per block-step.

4. RECURSIVE STATE-SPACE SYNTHESIS

In addition to the input-output approach in recursive layer peeling, there has also been a considerable interest in synthesis of lossless models from state-space descriptions. For instance, this approach has been used extensively in orthogonal synthesis of digital filters: starting with a given state space model for an allpass filter, one first obtains an *orthogonal state space model* of a particular (Hessenberg) form and then proceeds to obtain a realization in terms of elementary (2×2) lossless cells via a recursive factorization procedure (see, *e.g.,* Roberts and Mullis (1987)). Passive transfer functions can also be

realized in this fashion by embedding them into allpass filters with additional inputs and outputs. This can be accomplished either by spectral factorization, as in the method of Rao and Kailath (1984), or by solving certain Ricatti equations (see, *e.g.,* Desai(1989)).

To be more specific, denote the state space model associated with a given lossless cascade model (such as the one described in the previous section) by $\{\mathbf{A}_0, \mathbf{B}_0, \mathbf{C}_0, D_0\}$. This means that the transfer function of the lossless cascade $\{\Theta_0(z), \Theta_1(z), \ldots, \Theta_n(z)\}$ can be expressed in the form[2]

$$\Theta_0(z)\Theta_1(z) \ \ldots \ \Theta_n(z) = D_0 + \mathbf{C}_0(z^{-1}\mathbf{I} - \mathbf{A}_0)^{-1}\mathbf{B}_0 = D_0 + z\mathbf{C}_0(\mathbf{I} - z\mathbf{A}_0)^{-1}\mathbf{B}_0 \qquad (9)$$

Also, it can be shown that $\{\mathbf{A}_0, \mathbf{B}_0, \mathbf{C}_0, D_0\}$ is J-orthogonal, viz.,

$$\begin{pmatrix} \mathbf{A}_0 & \mathbf{B}_0 \\ \mathbf{C}_0 & D_0 \end{pmatrix} \begin{pmatrix} \mathbf{I} & 0 \\ 0 & J \end{pmatrix} \begin{pmatrix} \mathbf{A}_0 & \mathbf{B}_0 \\ \mathbf{C}_0 & D_0 \end{pmatrix}^* = \begin{pmatrix} \mathbf{I} & 0 \\ 0 & J \end{pmatrix} \qquad (10a)$$

where

$$J := \mathrm{diag}\{I_p, -I_q\} \qquad (10b)$$

Any other (minimal) realization, say $\{\mathbf{A}, \mathbf{B}, \mathbf{C}, D\}$, must be related to $\{\mathbf{A}_0, \mathbf{B}_0, \mathbf{C}_0, D_0\}$ via a similarity transformation, *i.e.,*

$$\mathbf{A} = \mathbf{T}^{-1}\mathbf{A}_0\mathbf{T}\,, \quad \mathbf{B} = \mathbf{T}^{-1}\mathbf{B}_0\,, \quad \mathbf{C} = \mathbf{C}_0\mathbf{T}\,, \quad D = D_0$$

It follows that the J-orthogonality relation (10) is now replaced by

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & D \end{pmatrix} \begin{pmatrix} \mathbf{R} & 0 \\ 0 & J \end{pmatrix} \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & D \end{pmatrix}^* = \begin{pmatrix} \mathbf{R} & 0 \\ 0 & J \end{pmatrix} \qquad (11)$$

where $\mathbf{R} := \mathbf{T}\mathbf{T}^*$. In particular we observe that

$$\mathbf{R} - \mathbf{A}\mathbf{R}\mathbf{A}^* = \mathbf{B}J\mathbf{B}^* \qquad (12)$$

which demonstrates the generalized displacement property of the matrix $\mathbf{R}$. This observation has led Genin et al. (1983) to propose a procedure for construction of lossless

[2]Recall that we use z instead of z^{-1} to denote a delay

cascade model by the state space approach and starting with the displacement equation (12). They have conjectured that their procedure gives rise to the same lossless cascade model as the one obtained via the Schur algorithm. However, this conjecture was never established.

The approach of Genin et al. (which is essentially the same as that of Roberts and Mullis) involves three steps:

(i) embedding eq. (12) into a state-space model $\{\mathbf{A}, \mathbf{B}, \mathbf{C}, D\}$ that satisfies the losslessness constraint (11).

(ii) transforming this state-space model into an equivalent *balanced form.*

(iii) factoring the balanced system matrix into a product of elementary (*i.e.* , single-state) balanced subsystem matrices, which corresponds to a cascade decomposition of the system described by $\{\mathbf{A}, \mathbf{B}, \mathbf{C}, D\}$.

We have recently constructed an alternative synthesis procedure that completely avoids the first two steps. Our procedure determines the same cascade realization as the one traditionally obtained by preliminary balancing, but it does so by (implicitly) factoring the *unbalanced model* $\{\mathbf{A}, \mathbf{B}, \mathbf{C}, D\}$ into a product of unbalanced single-state subsystems. Furthermore, only the matrices $\{\mathbf{A}, \mathbf{B}\}$ are required to carry out our procedure, which makes it possible to avoid the embedding step (i).

The only computation involved in our new procedure is a matrix recursion, viz.

$$\begin{pmatrix} 0 \\ \mathbf{G}_{i+1} \end{pmatrix} = \left\{ \mathbf{G}_i + (\mathcal{T}_i - I)\, \mathbf{G}_i \frac{J\beta_i^* \beta_i}{\beta_i J \beta_i^*} \right\} W_i \tag{13}$$

where $\mathbf{G}_0 \equiv \mathbf{B}$, β_i is the first row of $\mathbf{G}_i$, W_i is an *arbitrary* J-unitary matrix and $\mathcal{T}_i$ is determined (in a simple manner) by $\mathbf{A}$ alone (Fig. 6).

Figure 6. A Cascade Interpretation of the Computational Procedure (13).

This recursion is the matrix equivalent of the transfer domain recursion (5), which was originally derived without any reference to lossless state-space models. In fact, choosing $\mathbf{A} = \mathbf{L}(f)$ where $f(z) := \sum_{i=0}^{n} f_i z^i$ makes (13) the exact equivalent of (5), in the sense that the two are related via a suitable transform: when $\zeta_i = 0$ for all i this is the conventional Z-transform, but for non-vanishing ζ_i the appropriate transform is given by a *Newton-series* expansion (see Lev-Ari and Kailath (1990); Ackner, Lev-Ari and Kailath (1990b)).

The relation between recursive layer peeling and the factorization of matrices is now well understood (see Section 2, as well as Lev-Ari and Kailath (1984), Bruckstein and Kailath (1987)); it holds even for matrices with the generalized displacement representation (3), viz.,

$$d(\mathbf{Z}, \mathbf{Z})\mathbf{R} = \mathbf{G}J\mathbf{G}^* \tag{14}$$

where $\mathbf{G}$ denotes the matrix of coefficients of the power-series expansion of $G(z)$, (see, *e.g.,* Lev-Ari and Kailath (1986)). Furthermore, it follows that for every $d(z, w)$ of the form (4) and for every (finite) matrix $\mathbf{R}$,

$$In\,\{d(\mathbf{Z}, \mathbf{Z})\mathbf{R}\} = In\,\{d(\mathbf{Z}, \mathbf{Z})\mathbf{R}^{-\natural}\} \tag{15}$$

where the reversed matrix $\mathbf{R}^{-\natural}$ is obtained by transposing $\mathbf{R}^{-1}$ with respect to the anti-diagonal, namely

$$\mathbf{R}^{-\natural} := \tilde{\mathbf{I}}\,(\mathbf{R}^{-1})^T\,\tilde{\mathbf{I}} \tag{16a}$$

where the superscript T denotes the conventional (non-Hermitian) transpose, and $\tilde{\mathbf{I}}$ is the anti-diagonal unity matrix, viz.,

$$\tilde{\mathbf{I}} := \begin{pmatrix} & & & 1 \\ & 0 & . & \\ & & . & \\ & . & & 0 \\ 1 & & & \end{pmatrix} \tag{16b}$$

The fundamental result (15) implies that $\mathbf{R}$ and $\mathbf{R}^{-\natural}$ have the same displacement structure, and therefore that there exists a matrix $\mathbf{H}$ such that

$$d(\mathbf{Z}, \mathbf{Z})\mathbf{R}^{-\natural} = \mathbf{H}J\mathbf{H}^* \tag{17}$$

It does not tell us, however, how to obtain $\mathbf{H}$ or what is its corresponding cascade model. It has been shown in a limited context (and by fairly lengthy arguments) that the cascade model for $\mathbf{R}^{-\natural}$ is obtained by reversing the order of the layers in the cascade model for $\mathbf{R}$ (Lev-Ari and Kailath (1984)).

One of the advantages of the state-space approach is that it provides a direct connection between the representations of $\mathbf{R}$ and $\mathbf{R}^{-\natural}$. Indeed it follows directly from (11) that

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & D \end{pmatrix}^* \begin{pmatrix} \mathbf{R}^{-1} & 0 \\ 0 & J \end{pmatrix} \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & D \end{pmatrix} = \begin{pmatrix} \mathbf{R}^{-1} & 0 \\ 0 & J \end{pmatrix}$$

Therefore we can obtain an explicit characterization of the state-space model for $\mathbf{R}^{-\natural}$, viz.,

$$\begin{pmatrix} \widetilde{\mathbf{A}} & \tilde{\mathbf{B}} \\ \tilde{\mathbf{C}} & \widetilde{D} \end{pmatrix} \begin{pmatrix} \mathbf{R}^{-\natural} & 0 \\ 0 & J \end{pmatrix} \begin{pmatrix} \widetilde{\mathbf{A}} & \tilde{\mathbf{B}} \\ \tilde{\mathbf{C}} & \widetilde{D} \end{pmatrix}^* = \begin{pmatrix} \mathbf{R}^{-\natural} & 0 \\ 0 & J \end{pmatrix} \tag{18a}$$

where

$$\begin{pmatrix} \widetilde{\mathbf{A}} & \tilde{\mathbf{B}} \\ \tilde{\mathbf{C}} & \widetilde{D} \end{pmatrix} := \begin{pmatrix} \tilde{\mathbf{I}} & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & D \end{pmatrix}^T \begin{pmatrix} \tilde{\mathbf{I}} & 0 \\ 0 & I \end{pmatrix} \tag{18b}$$

Notice that $\widetilde{\mathbf{A}} \equiv \mathbf{A}^{\natural}$ and $\widetilde{D} \equiv D^T$. The corresponding transfer function is

$$\widetilde{D} + \tilde{\mathbf{C}}(z^{-1}\mathbf{I} - \widetilde{\mathbf{A}})^{-1}\check{\mathbf{B}} = D^T + \mathbf{B}^T\tilde{\mathbf{I}}(z^{-1}\mathbf{I} - \tilde{\mathbf{I}}\mathbf{A}^T\tilde{\mathbf{I}})^{-1}\tilde{\mathbf{I}}\mathbf{C}^T = [D + \mathbf{C}(z^{-1}\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}]^T$$

and, therefore,

$$\widetilde{D} + \tilde{\mathbf{C}}(z^{-1}\mathbf{I} - \widetilde{\mathbf{A}})^{-1}\tilde{\mathbf{B}} = \Theta_n^T(z)\ldots\Theta_0^T(z) \tag{19}$$

which establishes in a direct manner the fact that the cascade model for $\mathbf{R}^{-\natural}$ is obtained by reversing the order of the layers (and taking the transpose of each $\Theta_i(z)$) in the cascade model for $\mathbf{R}$. This makes it possible to reconstruct $\mathbf{C}$ from the cascade model obtained for $\{\mathbf{A}, \mathbf{B}\}$ and, consequently, to obtain explicit expressions for $\mathbf{R}^{-1}$ (*i.e.* , Gohberg-Semencul type formulas) as well as solutions of linear equations involving $\mathbf{R}$. In fact, our new cascade synthesis method gives rise to two new algorithms for the calculation of $\mathbf{C}$ for all matrices with a displacement structure. In the particular case of Toeplitz matrices one of these coincides with the well-known Levinson algorithm, while the other is entirely new.

The state-space approach has also been extensively used in the solution of the *Nevanlinna-Pick interpolation problem*, which arises, among other applications in the context of *model-order reduction*. The objective of model-order reduction is to approximate a given impulse response $\{h_k\}$ by a rational stable transfer function[3]

$$\widehat{H}(z) = \frac{b(z)}{a(z)} = \sum_{k=0}^{\infty} \hat{h}_k z^{-k} \tag{20}$$

of a prescribed order. It has been shown that when the quality of approximation is measured by the so-called Hankel norm (applied to the difference between h_k and $\hat{h}_k$) then $h_k - \hat{h}_k$ is the causal part of the impulse response of a certain non-causal filter that arises in the following interpolation problem (Genin and Kung (1981)):

> Given a stable and causal impulse response $\{h_k\}$ find a function $f(z)$ with r poles (or less) within the unit disc and with the least possible infinity-norm

[3]Here we shall maintain the convention that $\Theta_i(z)$ is a first-order matrix polynomial in z, while $\widehat{H}(z)$ is nevertheless defined as a power series in z^{-1}.

$||f||_\infty$ such that, for $i = 1, 2, \ldots, n$

$$f(\zeta_i) = \lim_{z \to \zeta_i} \{B(z)H(z)\}$$

where

$$H(z) = \sum_{k=0}^{\infty} h_k z^{-k}$$

is the transfer function associated with the given impulse response, $\{\zeta_i\}$ are the poles of this transfer function (all of which are within the unit disc) and $B(z)$ is the Blaschke product determined by these poles, *i.e.*,

$$B(z) = \prod_j \frac{z - \zeta_j}{1 - \zeta_j^* z} \tag{21}$$

The desired reduced-order transfer function $\widehat{H}(z)$ is obtained via

$$\widehat{H}(z) = H(z) - \text{causal part of } \left\{\frac{f(z)}{B(z)}\right\} \tag{22}$$

If $H(z)$ is a matrix function (corresponding to a MIMO system) then so are $\widehat{H}(z)$ and $f(z)$; in this case $\{\zeta_i\}$ are the Smith-MacMillan poles of $H(z)$.

Though $f(z)$ can be determined via the Nevanlinna-Pick procedure, it has become customary to solve this problem via the state space approach. Starting with a state-space model $\{A, B, C, D\}$ for the given impulse response one obtains first a balanced realization for the same transfer function and then, applying a transformation described by Glover (1984), determines the state-space model $\{\widehat{A}, \widehat{B}, \widehat{C}, \widehat{D}\}$ of the reduced-order transfer function $\widehat{H}(z)$.

The preference for working in the state-space domain has been, at least partly, motivated by the lack of efficient computational procedures for the *matrix* Nevanlinna-Pick problem. The standard procedure for solving this problem (see, *e.g.,* Delsarte, Genin, and Kamp (1979)) requires computationally-intensive matrix operations. The tangential Schur algorithm of Dewilde and Dym allows a significant reduction in computational requirements (though, strictly speaking it does not apply to the case of functions with

poles within the unit circle). A further simplification would be achieved if our tangential Schur algorithm can be generalized to arbitrary $\{\zeta_i\}$; currently it applies only to $\zeta_i = 0$ for all i. Since our procedure involves only scalar operations and determines the matrix function $f(z)$ directly from the given impulse response $H(z)$, it can provide an attractive alternative to the current state-space method for solving the model-order reduction problem.

5. WAVELET REPRESENTATION OF RANDOM PROCESSES

Wavelet representations of deterministic functions have recently become an area of active research in a variety of disciplines. Applications have been found in the fields of Mathematics (Mallat (1989)), Physics (Daubechies, Grossman and Meyer (1986)), Numerical Analysis (Glowinski et al. (1990)) and Signal Processing (Mallat (1989)). However, relatively few results are available for the properties of wavelet representations of random processes, the primary difficulty being that the existing theory of wavelets has been exclusively developed in the context of $L^2(\mathbb{R})$. We have shown that wavelet representations can be extended to random processes with either finite energy or finite power. We have also shown that *periodically-correlated* processes, namely processes with periodic statistics, give rise to wavelet representations with periodic correlation structures (Genossar, Goldburg, Lev-Ari and Kailath (1990)).

The wavelet representation at *resolution level* l of a function $f \in L^2(\mathbb{R})$ is

$$\hat{f}_l(t) = \sum_{k\in\mathbf{Z}} a_{0,k}\phi_{0,k}(t) + \sum_{m=0}^{l-1}\sum_{k\in\mathbf{Z}} b_{m,k}\psi_{m,k}(t), \quad l > 0 \ , \tag{23a}$$

where $\psi(t)$ is a *wavelet function* and $\phi(t)$ is a closely related function known as the *scaling function* associated with the wavelet $\psi(t)$. Dilations and shifts of these two functions, viz.,

$$\psi_{l,k}(t) := 2^{l/2}\psi(2^l t - kP) \ , \quad \phi_{l,k}(t) := 2^{l/2}\phi(2^l t - kP), \tag{23b}$$

give rise to a family of mutually orthonormal functions. Consequently, the coefficients of

the representation (23a) are given by inner products, viz.,

$$b_{l,k} := \langle f, \psi_{l,k} \rangle \quad \text{and} \quad a_{0,k} := \langle f, \phi_{0,k} \rangle. \tag{23c}$$

Convergence results

We have shown (in Genossar, Goldburg, Lev-Ari, and Kailath (1990)) that for wavelets with *finite support* the representation at resolution level l is well defined even for certain processes that are not members of $L^2(\mathbb{R})$ and, in particular, for processes with *finite power*. Moreover, the l-th level approximation $\hat{f}_l(t)$ converges to the original function in the following sense: For any compact interval $I \subset \mathbb{R}$,

$$\int_I |f(t) - \hat{f}_l(t)|^2 \, dt \longrightarrow 0, \quad \text{as } l \to \infty \ . \tag{24}$$

For wavelet representations of *random processes*, whether of finite energy or of finite power, we use the same definition as equation (23). Here too, we have shown that the coefficients and approximations are well defined. The corresponding convergence results are expressed in terms of mean square errors. For random processes with finite energy we have shown that

$$\lim_{l \to \infty} \int_{\mathbb{R}} \mathcal{E} \left\{ |x(t) - \hat{x}_l(t)|^2 \right\} \, dt = 0 \ . \tag{25}$$

For wavelet representations of random processes with finite power (and using wavelets of compact support) we have shown that for any compact interval I

$$\lim_{l \to \infty} \int_I \mathcal{E} \left\{ |x(t) - \hat{x}_l(t)|^2 \right\} \, dt = 0 \ . \tag{26}$$

Periodically-Correlated Processes

Periodically-correlated processes provide a particularly interesting example of wavelet representations for random processes. A periodically-correlated process with period P (also referred to as a cyclostationary process) is a random process $\{x(t) \, ; \ -\infty < t < \infty\}$

with finite second moments, whose mean and autocovariance functions satisfy the conditions:

$$\mathcal{E}\{x(t)\} = \mathcal{E}\{x(t+P)\} \tag{27a}$$

$$r(t,s) := \mathcal{E}\{x(t)x^*(s)\} = r(t+P, s+P) \tag{27b}$$

The wavelet representation of this process gives rise to a filter-bank model (Fig. 7).

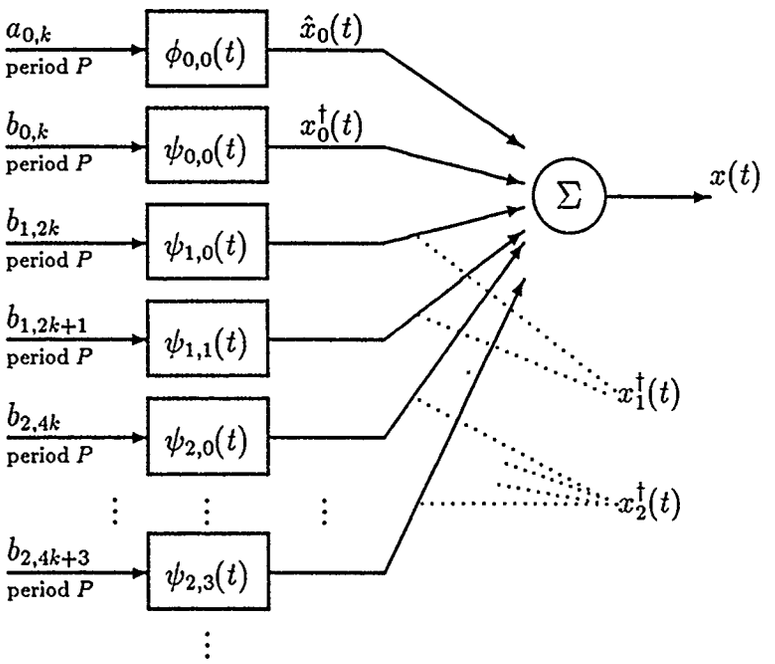


Figure 7. A Filter Model for Periodically-Correlated Processes

The summation of the outputs of the 2^l wavelet filters at resolution level l yields $x_l^\dagger(t)$, the detail signal at level l. We have shown that if $x(t)$ is a periodically-correlated process with period P, then all input sequences to this filter-bank model, namely the sequence $a_{0,k}$ as well as the sequences $b_{l,2^lk+r}$, $0 \le l$, $0 \le r \le 2^l - 1$ (for fixed l, r), are jointly wide-sense-stationary in the index k (see Genossar, Goldburg, Lev-Ari, and Kailath (1990)).

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1. T. Kailath, "From Kalman Filtering to Innovations, Martingales, Scattering and Other Nice Things," *Mathematical System Theory: The Influence of R. E. Kalman*, reprinted in *Communications and Control Engineering.*

2. Y. Bistritz, H. Lev-Ari and T. Kailath, "Immittance-Domain Three-Term Schur and Levinson Recursions for Quasi-Toeplitz Complex Hermitian Matrices," *SIAM J. on Matrix Analysis and Appls..*

3. A. Bruckstein, T. Kailath, I. Koltracht and P. Lancaster, "On The Reconstruction of Layered Media From Reflection Data," *SIAM J. Matrix Analysis.*

4. H. Lev-Ari, Y. Bistritz and T. Kailath, "Generalized Bezoutians and Families of Efficient Root-Location Procedures,' *IEEE Trans. Circ. Syst.*

Conference Papers.

1. T. Kailath and J. Chun, "Generalized Gohberg-Semencul Formulas," *International Conference on Operator Theory: Advances and Applications*, plenary lecture, Calgary, Canada, August 22-26, 1988. Reprinted in *Advances and Applications*, dedicated to I. Gohberg, *Gohberg Anniversary Volume*, Vol. 40, pp. 231-246, Springer-Verlag, 1989.

2. H. Lev-Ari, Y. Bistritz and T. Kailath, "Generalized Bezoutians and Efficient Root-Location," *8th Inter'l. Symposium on The Malth. Thy. of Networks and Systems*, Phoenix, AZ, June 15-19, 1987. Reprinted in *Linear Circuits, Systems and Signal Processing: Theory and Application*, ed. C. I. Byrnes, C. F. Martin, R. E. Saeks, pp. 63-72, North Holland, New York, 1988.

Papers Under Review.

1. R. Ackner, H. Lev-Ari and T. Kailath, "The Schur Algorihtm for Scalar Meromorphic Functions," *IEEE Trans. Circuits and Systems.*

2. H. Lev-Ari, K-F. Chiang and T. Kailath, "Constrained-Input/Constrained-Output Stability for Adaptive RLS Lattice Filters," *IEEE Trans. Circuits and Systems.*

3. H. Lev-Ari, T. Kailath and J.Cioffi, "Adaptive Recursive-Least-Squares Lattice and Transversal Filters for Continuous Signal Processing," IEEE Circuits and Systems.

4. A. Dembo and O. Zeitouni, "Maximum A-Posteriori Estimation of Random Fields - Part II: Non-Gaussian Fields," *Journal Multivariate Analysis.* J. Chun and T. Kailath, "Systolic Array Implementation of the Square Root Chandrasekhar Filter," *IEEE Trans. on Automa. Control.*

Keynote and Plenary Lectures At Scientific/Technical Society Conferences.

1. Centennial Lecturer, 1988 SIAM Annual Meeting, Minneapolis, MN, July 1988.

2. Keynote Lecture, NATO Advanced Study Institute on Signal Processing and Parallel Computing, Leuven, Belgium, August 1988.

3. Symposium on Applied Mathematics and Scientific Computing, Computer Science Department, Stanford University, April 21, 1989

4. SDIO/IST Workshop on Sensor Signal Processing, Washington, D.C., April 24-27, 1989

5. Special Lecture Series, Imperial College of Science Technology and Medicine, Department of Electrical Engineering, London, England, Summer 1989

6. Plenary Lecture, International Symposium on the Mathematical Theory of Networks and Systems (MTNS-89), Amsterdam, The Netherlands, June 22, 1989

7. Keynote Lecture, International Symposium on Systems Engineering, Wright State University, Dayton, OH, August 23, 1989

8. Invited Lecture, Math. Assoc. of America Annual Meting, Louisville, KY, January 1990.

Honors and Awards.

- First Hitachi American Professor of Engineering, Stanford University, February 1988.
- 1988 Electrical Engineering Distinguished Service Award, Stanford University, June 1988.
- 1989 Technical Achievement Award of the IEEE Acoustics, Speech and Signal Processing Society
- Royal Society Guest Research Fellowship, Imperial College of Science, Technology and Medicine, Department of Electrical Engineering, London, England, Summer 1989
- Degree of Doctor Honoris Causa, Linkoping Institute of Technology, June 1990.

Advanced Degrees.

- D. Pal, "Fast Algorithms for Structures Matrices With Arbitrary Rank Profile," Department of Electrical Engineering, Stanford University, May 1990.
- J. Chun, "Fast Array Algorithms for Structured Matrices," Department of Electrical Engineering, Stanford University, June 1989.
- D. T. M. Slock, "Fast Algorithms for Fixed-Order Recursive Least-Squares Parameter Estimation Department of Electrical Engineering, Stanford University, September 1989.

Funding Category

Research Time

			Summer 1988 & 1989	Academic '88, '89 & '90
PI	Thomas Kailath		3 months (20%)	9 months (12%)
Graduate Students	R. Ackner	Israel	3 months (100%)	3 months (50%)
	T. Wang	China	0	3 months (50%)
Research Associate	H. Lev-Ari	Israel	1 month (50%)	4 months (30%)
Visiting Research Associate	B. Demoor	Belgium		3 months (50%)